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Active damping of a vibrating string.

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ABSTRACT

This paper investigates the active damping of the vertical and horizontal transverse modes of a rigidly-terminated vibrating guitar string. After describing the characteristics of various actuators and sensors, we motivate the choice of collocated electromagnetic actuators and a multi-axis piezoelectric bridge sensor. Next, we introduce a state-space model that emulates the behavior of the string, and we explain the theory behind band pass filter control and PID control as applied to a vibrating string. Finally, we discuss the experimental results and implications of applying these control methods. In particular, we consider the difference between damping the energy in only one transverse axis, versus damping the energy in both the vertical and horizontal transverse axes simultaneously.

1 INTRODUCTION

The field of “active instruments” is the study of actively controlling the vibrating structures in a musical instrument in order to alter its behavior [1]. Although it is possible to design the instrument from beginning to end in consideration of the control aspects, we choose to apply control to traditional instruments. This means that more musicians will be available to masterfully play the active instruments, but it also means that the control task is more challenging because some aspects of the instrument may be non-ideal from a control perspective.

In this paper, we choose to limit the effects of the instrument body and focus on the vibrating guitar string itself. This forms a solid starting point because the musician interacts most intimately with the string and only tangentially with the body. In addition, since canceling vibration is generally more difficult than inducing vibration, we jump directly to the heart of the matter by focusing on a detailed study of actively damping the string. Furthermore, while damping the string, we strive to preserve the musical qualities of the string that have been optimized during the evolution of stringed instruments.

From a practical standpoint, we also wish for the damping to be independent of the fundamental frequency (the lowest resonance) f_0 . That is, for better control of the active instrument by the musician, we wish for the damping parameter to be orthogonal to the string length, which is adjusted by the musician during play.

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2 PRIOR WORK

2.1 “Infinite” Sustain

Various forms of active instruments have already been designed and played. For instance, the inverse problem of indefinitely sustaining string vibration has long been investigated, especially in the framework of electric guitars. Musicians have used acoustic feedback from power amplifiers to re-excite their electric guitar strings thus producing sustain; however, due to the complex nature of the transfer functions involved and the nonlinear nature of the amplifiers, this has been difficult to control precisely. The commercially-available EBow [2] and Sustainiac [3] have mitigated this problem using phase-locked-loops (PLLs). Similarly, Weinreich and Caussé have electromechanically induced the Helmholtz “stick-slip” bowing motion in a vibrating string in the absence of a traditional bow [4].

2.2 Active Structural Control

Active control has also been applied to instrument body structures. For example, in order to suppress tendencies toward acoustic feedback in amplified situations, Griffin has altered the behavior of the plate modes in an acoustic guitar [5]. Similarly, Hanagud has modified the plate modes in an inexpensive acoustic guitar to make it behave more like an expensive acoustic guitar [6]. In addition, Charles Besnainou has tuned the Helmholtz body resonance of a guitar using PID control [7].

2.3 Active Instruments

Besnainou has also applied control to the violin, a snare drum, a pipe organ, and a marimba bar, and he has coined the term “Active Instruments” [1][8]. For example, he has changed the damping time and pitch of a marimba bar using PID control. He may have chosen the marimba bar in particular because the second-to-lowest resonance is normally tuned to almost two octaves above the fundamental frequency (lowest resonance), and each bar has a fixed fundamental frequency [9]. This simplicity makes the instrument much easier to control. Besnainou has carried out additional work on other instruments, but much of this work has remained unpublished.

More recently, Rollow has written his PhD thesis on controlling the vibrations of an air-loaded drum head [10], and Maarten van Walstijn and Rebelo have used various filtering techniques to alter the modes of a conga drum [11].

3 OVERVIEW

3.1 Physics Of A Vibrating String

Primarily transverse waves govern the musical characteristics of a vibrating string. When the string is actuated transversely in only the vertical y -axis, transverse waves arise exclusively in the y -axis along the string. Standing waves result when the string is terminated in at least two points. The standing waves can be any linear combination of vibrations at the harmonics of the fundamental frequency f_0 .

In general, however, strings on musical instruments vibrate in both the vertical and horizontal transverse axes because musicians actuate the string in both axes, and non-ideal string terminations allow an exchange of energy between the horizontal and vertical transverse axes. In addition, various nonlinear effects can cause additional

exchanges of energy between the transverse axes and even energy exchanges between the harmonics [12].

3.2 Control Configuration

In order to control the transverse waves in a vibrating string, both the vertical and horizontal axes need to be sensed and actuated. Although there is some exchange of energy between the axes, any robust control strategy should allow these exchanges to be neglected. Since ideally the axes are independent of each other, we can design identical controllers for each axis. Figure 1 shows a block diagram for controlling the vertical axis. $K(s)$ represents the transfer function of the controller, $Gain$ represents the amount of control used, and $G(s)$ represents the transfer function of the vibrating string between the actuator and sensor. Note that $G(s)$ may be time-varying as the musician may change the length of the string in order to adjust f_0 .

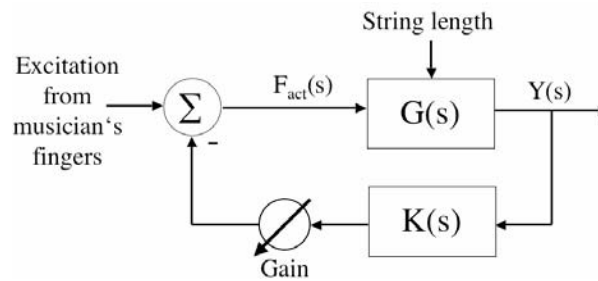


Figure 1: System block diagram for the vertical axis.

3.3 Actuators

One might imagine that actuating the string with a piezoelectric device would be feasible. However, piezoelectric bending elements have much smaller impedances than a guitar string, and piezoelectric stacks have much larger impedances than a guitar string. Surprisingly, there are no widely-available piezoelectric-based actuators that are matched to a guitar string's impedance. As a result, we chose to use electromagnetic actuators [13]. Due to the slight stiffness of the string, these actuators are inefficient when mounted too closely to the termination; however, they behave too nonlinearly when mounted too far from the termination, so a compromise is needed.

3.4 Sensors

Many different kinds of sensors can be used to detect the string's motion. Electromagnetic sensors are traditionally used in electric guitars, but they are large and behave more nonlinearly than other string sensors [14]. In contrast, piezoelectric sensors are smaller and linear, but they must usually be mounted at a string termination. Sensing both the vertical and horizontal transverse axes with piezoelectrics is more complicated, but the Center for New Music and Audio Technologies (CNMAT) at UC Berkeley was kind enough to lend us a special piezoelectric multi-axis string sensor, which we used to terminate the string [14]. Optical sensors involving infrared emitter and detector pairs are quite common, and so we used a pair of these to calibrate the multi-axis string sensor [12].

3.5 Actuator/Sensor Placement

Since collocated control is generally more robust, we chose to mount the sensors and actuators as closely together as possible at the bridge (see Figure 2). In this manner, the musician may change the length of the string by moving the opposite string termination (not shown). A pair of electromagnetic actuators was placed 4cm to the left of the string termination (see Figure 2a) to make for a good compromise between efficiency and linearity. A pair of optical sensors was mounted 1cm from the string termination in order to record the motion of the string (see Figure 2b). Finally, the string was terminated with the piezoelectric multi-axis string sensor from CNMAT (see Figure 2c). The piezoelectric sensor was used to close the control loop as it was less prone to noise than the optical sensors. All sensors and actuators were mounted such that the horizontal and vertical axes could be sensed and actuated independently of each other.

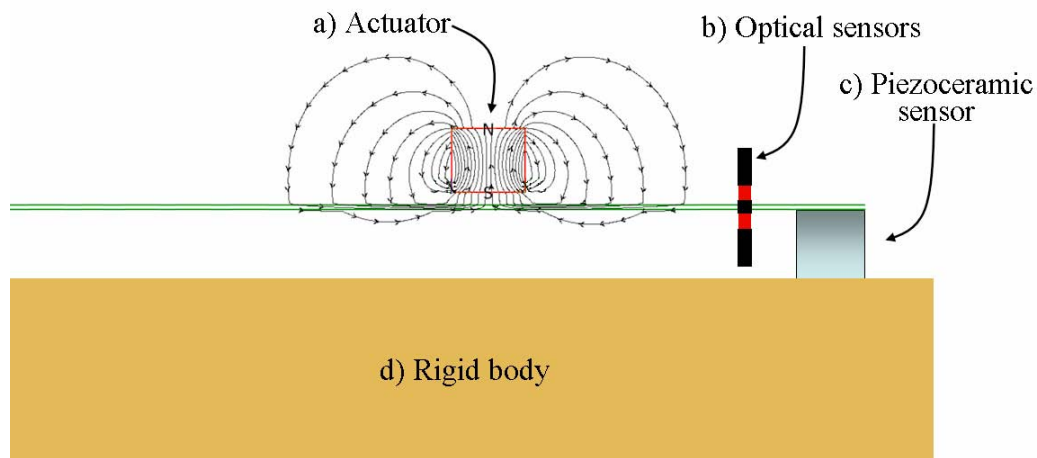


Figure 2: Diagram of one end of the string showing the actuator and sensor placement at the bridge.

3.6 Control Strategy

In order to actively damp the string, we may simply drive the string's displacement to zero at a point arbitrarily close to the bridge. This is equivalent to decreasing the resistive component of the bridge termination's effective impedance. We must however take care to avoid making the bridge termination impedance complex. This would result in detuning the harmonics of the string, which would conflict with our goal of preserving the sound quality of the instrument [9].

3.7 State-Space Model

The simplest state-space model of the string incorporates only the lowest resonance at f_0 . Figure 3 shows the physical diagram of a simple system with one resonance, where $y(t)$ is the displacement of the mass, and $f_{act}(t)$ is the force exerted on the mass.

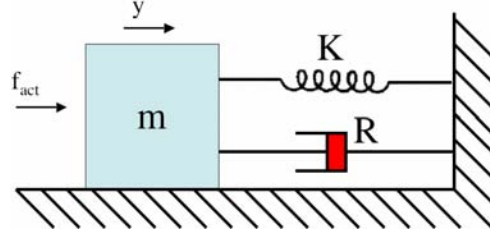


Figure 3: A simple mechanical oscillator with damping.

The behavior of the system can be easily represented in the Laplace domain.

$$(ms^2 + Rs + K)Y(s) = F_{act}(s) \quad (1)$$

By matching the decay time constant $\tau = 1/\alpha = 2m/R$ (in sec) and $\omega_0 = \sqrt{K/m}$ (in rad/sec) to those of the lowest resonance of a vibrating string, we can build a simple model of a vibrating string since the lowest resonance usually has the longest decay time. The transfer function of the linear system between the input force and the output displacement is as follows:

$$\frac{Y(s)}{F_{act}(s)} = \frac{1}{m} \frac{1}{s^2 + 2\alpha s + \omega_0^2} \quad (2)$$

If we consider for example the $N = 10$ lowest resonances, then we can build a more accurate model. Since the modes are ideally independent of each other, the resonances are placed in parallel.

$$G(s) = \frac{Y(s)}{F_{act}(s)} = \sum_{n=1}^N \frac{1}{m_n} \frac{1}{s^2 + 2\alpha_n s + (n\omega_0)^2} \quad (3)$$

4 CONTROL ALGORITHMS

4.1 Band Pass Filter Control

Before investigating classical control methods, it is worth mentioning that there is a simple controller that can be built using standard audio hardware despite its inherent system delays. For example, our implementation includes a personal computer, a standard audio interface, and the signal processing environment for computer music known as Max/MSP. It should be noted that this control method is particularly useful for testing the controllability and the role of nonlinearity in a highly-resonant system. The block diagram for the controller is shown in Figure 4.

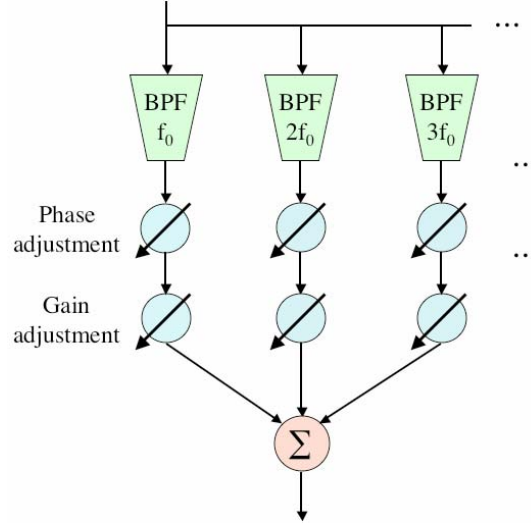


Figure 4: Block diagram for the controller $K(s)$ consisting of a bank of band pass filters.

The controller uses precisely-tuned band pass filters to isolate the energy in each of the harmonics. The phase of the output from each filter can be adjusted manually so that the associated mode either grows or decays over time. Finally, the gain adjustment controls how fast the mode grows or decays. The controller is highly sensitive to the fundamental frequency f_0 and the quality factor Q of the filters. We found that $Q = 30$ made for a good compromise between sensitivity and isolation of the modes.

4.2 PID Control

On the other hand, classical controllers are much more appealing since they are independent of f_0 . This makes them much more useful in a performance context, even though individual modes cannot be easily independently controlled. The analysis is simple for controlling a single resonance using the proportional, integral, and derivative functions of the velocity. For unity with the following sections, we will equivalently investigate the second derivative (P_{DD}), the derivative (P_D), and the proportional (P_P) functions of the displacement $y(t)$.

$$(ms^2 + Rs + K)Y(s) = F_{act}(s) = (P_{DD}s^2 + P_Ds + P_P)Y(s) \quad (4)$$

$$[(m - P_{DD})s^2 + (R - P_D)s + (K - P_P)]Y(s) = 0 \quad (5)$$

The solution is of the following form:

$$y(t) = C_0 e^{-\hat{\alpha}t} \cos(\hat{\omega}_0 t + \hat{\phi}) \quad (6)$$

The frequency of vibration $\hat{\omega}_0$ is affected by P_P and P_{DD} .

$$\hat{\omega}_0 = \sqrt{\frac{K - P_P}{m - P_{DD}}} \quad (7)$$

Similarly, the damping rate $\hat{\alpha}$ is affected by P_D and P_{DD} . Thus, P_D could be used to control the decay rate independently of the frequency. This is the kind of orthogonal control that we seek.

$$\hat{\alpha} = \frac{R - P_D}{2(m - P_{DD})} \quad (8)$$

4.2.1 Derivative Control

Since a vibrating string actually has multiple resonances, it is instructive to create the root locus for the P_D term (see Figure 5) using the state-space model. The magenta squares show the pole locations when a moderate amount of control is applied, such that the damping time is about the same as achieved in the experimental section. As expected, the additional damping is roughly independent of the frequency of the harmonic.

One might also be concerned that the truncated order of the model would drastically change the locus' characteristics, but since each resonance of the string is accompanied by a nearby anti-resonance, each harmonic only affects the locus locally.

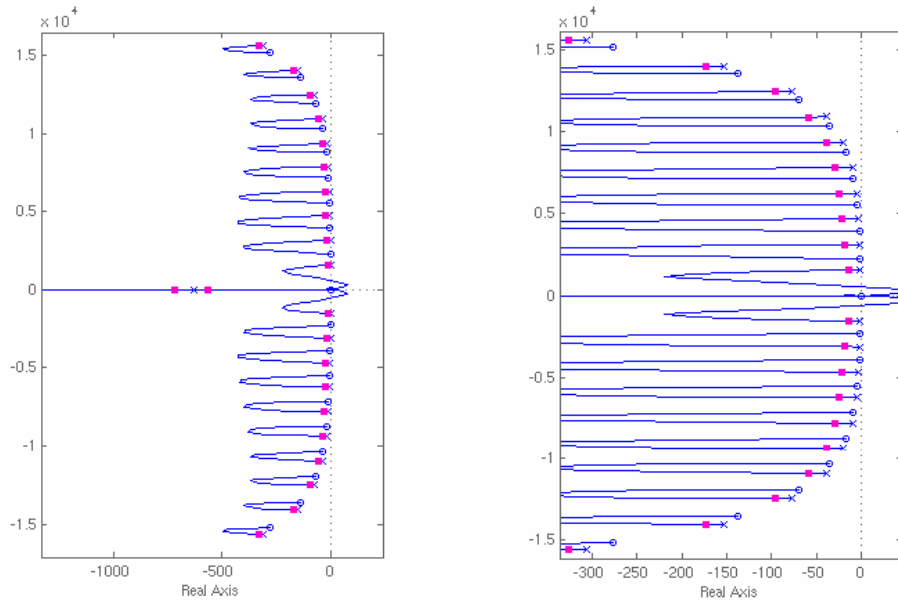


Figure 5: 180° Root locus (left: zoomed out, and right: zoomed in) for derivative control.

Since the lower harmonics of the uncontrolled string have less damping, we might optimally wish to invest more control energy in damping the lower harmonics.

4.2.2 Integral Control

By applying integral control of the displacement (P_I), we can invest a larger fraction of the control energy in damping the lower harmonics.

$$(ms^2 + 2\alpha ms + K)\tilde{Y}(s) = F_{act}(s) = \frac{1}{s}P_I\tilde{Y}(s) \quad (9)$$

An approximate derivation of the effects of P_I can be made by assuming that $|P_I|$ is small enough such that the solution is approximately the same as when $P_I = 0$ except for the damping coefficient $\tilde{\alpha}$:

$$\tilde{y}(t) = C_0 e^{-\tilde{\alpha}t} \cos(\sqrt{\omega_0^2 - \tilde{\alpha}^2}t + \phi) \quad (10)$$

Further simplifications can be made since $\omega_0 \gg \tilde{\alpha}$ as long as the resonance is not being damped especially quickly. For example, if $\tilde{\alpha} = 1/\tilde{\tau} = 10 \text{ sec}^{-1}$ for a fairly fast decay time of 1/10 sec, while $\omega_0 > 2\pi(248) \approx 1560 \text{ sec}^{-1}$, then we certainly have $\omega_0 \gg \tilde{\alpha}$. After substituting $\tilde{y}(t)$ into the time-domain version of the dynamics differential equation (9), simplifying, and solving for $\tilde{\alpha}$, the following approximation can be obtained in terms of the passive damping rate α :

$$\tilde{\alpha} \approx \frac{R + P_I / \omega_0^2}{2m} = \alpha + \frac{P_I}{2m\omega_0^2} \quad (11)$$

Thus, for moderately-sized $\tilde{\alpha}$, the increase in the damping rate decreases with $\omega_0^2 = K/m$. Integral control applied to the string model results in the 0° locus shown in Figure 6. As expected the lower harmonics are damped more than the higher harmonics.

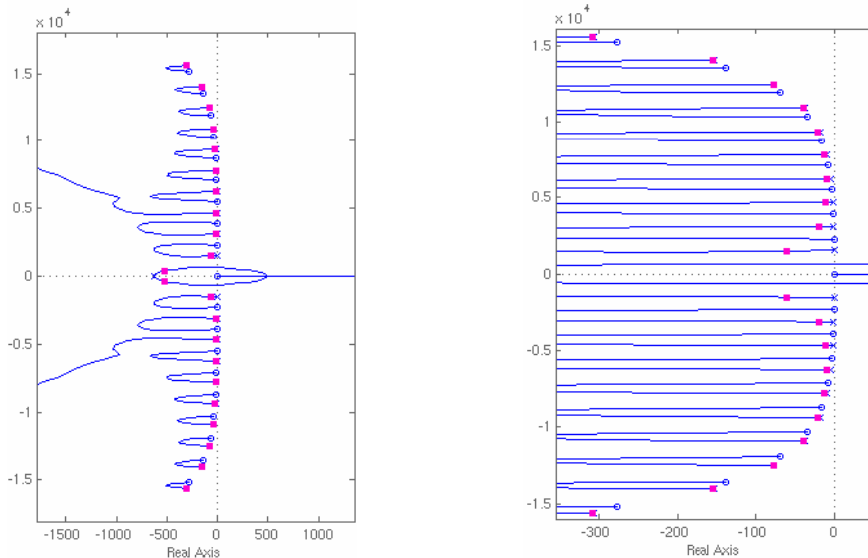


Figure 6: 0° Root locus (left: zoomed out, and right: zoomed in) for integral control.

One additional advantage of integral control over derivative control is the low-pass characteristic of integration, which makes the controller more robust to system delays. Theoretically, in order to damp the string as quickly and efficiently as possible, a combination of integral and derivative control would perform better than derivative or integral control alone; however, we will see that for the given experimental system, only integral control will be suitable due to system delay.

5 EXPERIMENTAL RESULTS

We carried out several experiments in order to learn about the challenges one would encounter in building the active instrument. The results from the most successful experiment will be explained here.

In order to reduce the effects of the body, we mounted the string on an 8.9cmx8.9cm (3½"x3½") block of hardwood ash, and the string terminations were made as rigid as possible. The string itself was an E electric guitar string (1.1mm in diameter) with a steel core and wound with nickel. We powered the actuators with a 100W/channel Marantz audio power amplifier, and because the current-output of the amplifier behaved roughly like an integrator above 200Hz, we shortened the string to approximately 24cm. The fundamental frequency then became $f_0 = 248$ Hz.

Derivative control was found to be unsuitable for the experiment due to system delay—the actuator and sensor were not exactly collocated. In addition, the actuator was placed 4cm from the string termination, and so it was near a node for the 6th, 12th, 18th, ... harmonics. As a result, only the four or five lowest modes were damped using integral control, but this proved quite sufficient in greatly reducing the damping time of the string.

5.1 Dual-Axis Cancellation

Since the active instrument was designed to be played by a musician, the string was plucked manually by hand in tests. For example, Figure 7 depicts four actively-cancelled plucks followed by one pluck without control. In particular, the top waveform in Figure 7 shows the horizontal displacement of the string at the bridge, while the bottom waveform in Figure 7 shows the vertical displacement.

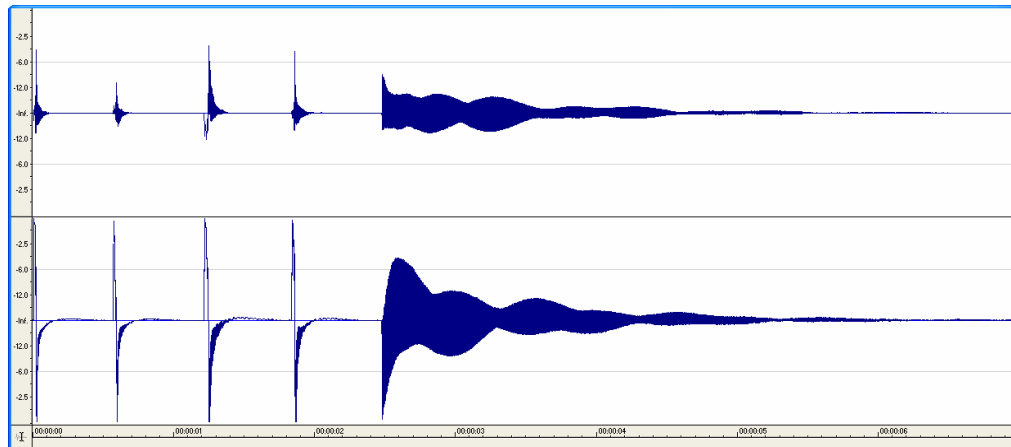


Figure 7: Waveforms displaying four cancelled string plucks and then one pluck without control. (top: horizontal axis, bottom: vertical axis)

Spectrograms of the same waveforms were generated and displayed in Figure 8. The decay times of the lowest four modes were greatly reduced, while the decay times of the higher modes remained approximately constant. The small differences in the plucks resulted from the fact that the string could not be plucked perfectly consistently by hand, while the plucks were nevertheless of roughly the same magnitude.

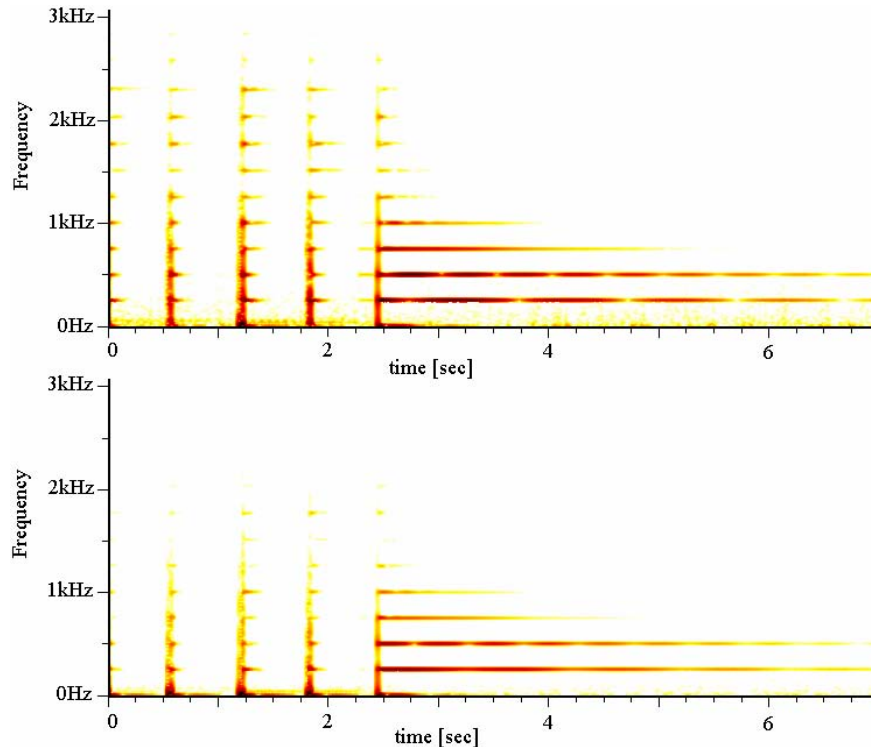


Figure 8: Spectrograms displaying four cancelled string plucks and then one pluck without control. (top: horizontal axis, bottom: vertical axis)

The behavior was consistent and independent of fundamental frequency. Active damping required less than 1 Watt according to the power meters on the Marantz amplifier. By varying the loop gain, it was of course possible to vary the damping over a large range of intermediate values.

5.2 Single-Axis Cancellation

Theoretically, if the vibrations in the vertical axis were cancelled, while no control were applied to the horizontal axis, then the two axes would behave independently. However, in practice the sensors and actuators cannot be perfectly aligned, and so there will be some exchange of energy between the axes. It follows that single-axis cancellation is a good test of sensor/actuator alignment.

Figure 9 shows the waveforms of the string displacement resulting from the following experiment during which the string was plucked by hand repeatedly at roughly the same magnitude: at the beginning (the left side of the waveforms), control was applied to only the vertical axis. Then as time progressed, control was added to the horizontal axis. Near the midpoint, the amounts of control were equal, after which time the control applied to the vertical axis was slowly reduced to zero.

It can be seen that the vertical actuator and sensor were well aligned—at the beginning of the waveforms, the vertical axis remains largely cancelled even when the horizontal axis is not cancelled. On the other hand, the horizontal actuator and sensor were not perfectly aligned as the horizontal axis was only well-damped when both the vertical and horizontal axes are damped. This was due to the fact that the piezoelectric sensor did not measure the horizontal axis as accurately as it measured the vertical axis.

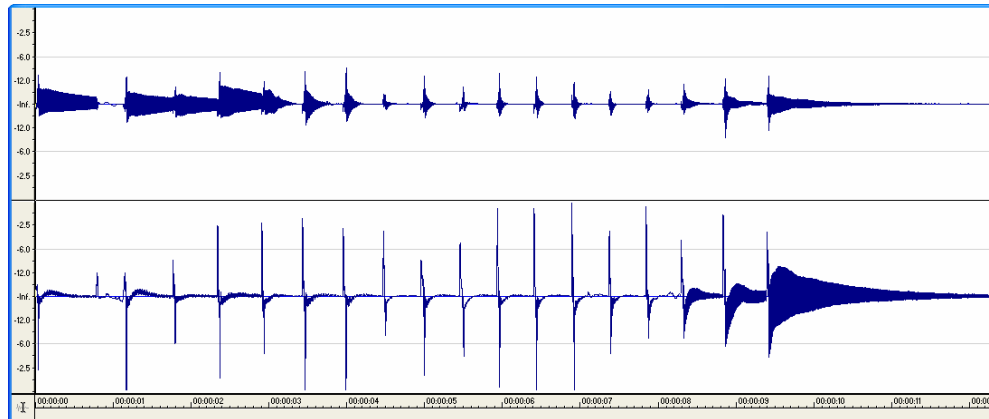


Figure 9: Waveforms displaying cancelled string plucks as the controlled axes are gradually varied. (top: horizontal axis, bottom: vertical axis)

6 CONCLUSION

We have reviewed the prior work on active instruments and discussed the advantages and drawbacks of various string actuators and sensors. Then, after developing a model of the string, we have discussed band pass filter control, which could control the modes independently but was dependent on the fundamental frequency f_0 of the string. In contrast, PID control of string displacement did not depend on f_0 and could be used to damp the string quickly. Finally, we have presented waveforms of actively damped string vibrations and demonstrated that the sensors and actuators must be carefully aligned in order to obtain independent control of the vertical and horizontal transverse axes.

Future work will likely involve exploring the complete PID(DD) space as well as implementing various sound effects for electric guitar directly upon the strings themselves. For example, the tremolo effect consists of periodically varying the amplitude of an input signal, and so the active damping parameter could simply be varied periodically to approximately achieve this effect.

7 WAVEFORMS

The waveforms corresponding to the model, the magenta poles displayed in the root loci, and the experiments are available on the internet. They can be downloaded from the website: <http://ccrma.stanford.edu/~eberdahl/Projects/ActiveDamping/index.html>

8 ACKNOWLEDGEMENTS

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